Supplementary material to Crypto Premium, Higher-Order Moments and Tail Risk

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This version: September 6, 2022

This document is organized as follows:

- Section 1 presents empirical evidence on the relationship between the estimates for the crypto premium and the estimates for alternative SDF of cryptocurrency investors proposed in the literature.
- Section 2 presents a descriptive statistics and further details on the sample of the cryptocurrency used in the construction of the the portfolios.
- Section 3 presents the risk premium estimates for alternative asset classes, like equity, fixed income and commodity.
- Section 4 presents the properties of portfolios for different sorts.
- Section 5 presents details on the empirical strategy used to estimate the market prices of risks.

This Appendix is available on the authors' websites.

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1 Crypto premium and crypto factors

In this section we show that the crypto premium and its components are significantly related to alternative specifications of the SDF of cryptocurrency investors that have been used in the literature. Specifically, we find that the crypto premium is related to the $Carry_{Btc}$ factor from Borri and Shakhnov (2021), which shows that the covariance with the latter factor can explain the positive excess returns associated with strategies based on bitcoin price differences across exchanges.

Table 1 presents the results of linear regressions of the two factors used in Borri and Shakhnov (2021) on the crypto premium and its components. The first factor, $Carry_{Btc}$, is the return spread between the portfolio invested in the bitcoin pairs with the largest discounts and the portfolio invested in bitcoin pairs with the lowest discounts. Bitcoin discounts are a measure of the bitcoin price differences across exchanges and currency pairs. The second factor, Btc_{Kraken}, is a bitcoin factor similar to the dollar factor from the FX literature, and goes long all cross-exchange portfolios. The regression equations we estimate are as follows:

$$Fact_{t+1} = cons + \sum_k \beta^k X^k_{t+1|t} + \epsilon_{t+1}$$

where $Fact_{t+1}$ can be either $Carry_{Btc,t+1}$ or $Btc_{Kraken,t+1}$; X^k can be equal either to $m_{t+1|t}$ or to the components of the crypto premium, that is $Var_{t+1|t}, Sk_{t+1|t}, Kurt_{t+1|t}, \lambda_{t+1|t}$. Note that the regressors are conditional expectations at t of the value of a given variable at $t+1$. The dependent variables are in the space of returns, and are values realized at $t+1$. We can think of the latter as the expectation of the factor plus an error term (e.g., $Carry_{Btc,t+1} = E_t[Carry_{Btc,t+1}] + \epsilon_{t+1}$).

The table shows that both $Carry_{Btc}$ and Btc_{Kraken} are significantly positively related to the crypto premium: that is, expected returns associated with the two cross-exchange strategies are, on average, higher when investors expect higher bitcoin excess returns. Borri and Shakhnov (2021) show that Btc_{Kraken} is a level factor, on which portfolios load similarly and, thus, does not carry a premium. On the contrary, $Carry_{Btc,t+1}$ is a slope factor with a positive premium. We find that the crypto premium is mostly related to the latter factor, as exemplified by the R-squared of about 20%. Looking at the components of the crypto premium, we find that investors expect higher $Carry_{Btc}$ returns when they also expect lower skewness and higher kurtosis associated with bitcoin returns, and a lower jump intensity.

2 Cryptocurrency Data

This section presents descriptive statistics related to the cryptocurrencies used in the construction of the portfolios used in the estimation of the asset pricing model of Section 4.5. Table 2 presents descriptive statistics of the sample of cryptocurrencies. The number of available currencies in our sample increases from 574 in 2017 to 852 in 2021. The mean (median) market capitalization in the sample is 904.79 (5.83) million dollars. The mean (median) daily trading volume is 466.78 (2.22) thousand dollars. We construct a cryptocurrency market return as the market capitalization weighted return of all the available currencies. Because data on market capitalization is available only for a subset of currencies, we also construct a volume-weighted market return. We note that although it has been documented that cryptocurrency volume data could be unreliable (at least for some exchanges, see for instance the report Bitwise, 2019), the sample correlation coefficient between value- and volume-weighted market returns is large and equal to 0.91.

Table 1: Crypto premium and crypto factors

	$Carry_{Btc}$	$Carry_{Btc}$	Btc_{Kraken}	Btc_{Kraken}
Panel A: Crypto premium				
$m_{t+1\|t}$	0.026^{a}		0.012^{a}	
	[0.002]		[0.003]	
Panel B: Crypto premium components				
$Var_{t+1\|t}$		−0.000		0.000
		[0.000]		[0.000]
$Sk_{t+1\|t}$		-5.550^{a}		−1.361
		[0.630]		[1.015]
$Kurt_{t+1\|t}$		0.009^{a}		0.001
		[0.001]		[0.002]
$\lambda_{t+1\|t}$		-0.008^{c}		-0.018^{b}
		[0.004]		[0.007]
$cons$	−0.000	0.025^{a}	0.000	0.020^{a}
	[0.001]	[0.004]	[0.001]	[0.007]
R^2 (%)	19.968	20.837	1.902	2.350

Notes: The table presents the estimates from linear regressions of the $Carry_{Btc}$ and Btc_{Kraken} factors from Borri and Shakhnov (2021) on the crypto premium and its components. Data are daily, from Cryptocompare. The sample period is January 1 2017 to December 31 2020.

3 Risk premia for non-crypto assets

This section studies the determinants of the risk premia obtained by estimating our model on different asset classes. We follow the same empirical strategy presented in Section 4.5, and relate the risk premia to the same set of crypto and non-crypto determinants. In particular, we estimate the following set of regressions

$$\Delta m^i_{t+1|t} = \alpha^i + \sum_{j=1}^{J} \beta^i_j F^j_t + \epsilon^i_t, \tag{1}$$

where Δ is the first difference operator (i.e., $\Delta m^i_{t+1|t} = m^i_{t+1|t} - m^i_{t|t-1}$), and the index i denotes the different asset classes. Table 3 presents the main results. The contribution of the crypto factors is either non-significant, or quantitatively very small. In contrast, the contribution of the non-crypto factors explains a large fraction of the U.S. equity risk premium (Mkt_{US}) and of the first principal components extracted from the risk premia of all assets (PC_1).

Table 2: Summary statistics

Panel A					
Year	Number of coins	Market cap (mil)		Volume (thous)	
		Mean	Median	Mean	Median
2017	574	1778.05	21.19	882.62	5.62
2018	504	210.92	3.60	177.89	1.17
2019	464	255.56	1.54	176.65	0.99
2020	788	1049.50	2.11	711.71	4.48
2021	852	2763.39	4.43	1919.59	13.23
Full	3319	904.79	5.83	466.78	2.22

Panel B					
	Mean	Median	Std	Skewness	Kurtosis
market return (value-weighted)	0.43	0.35	3.93	-0.32	8.42
market return (volume-weighted)	0.93	0.72	4.89	-0.20	7.93
bitcoin return	0.33	0.22	4.15	-0.14	9.83
ethereum return	0.55	0.00	7.82	2.49	22.13
ripple return	0.47	0.10	5.79	0.18	8.14

Notes: This table reports summary statistics for the cryptocurrencies in our sample. Panel A reports the number of coins, the mean and median of market capitalization, and the mean and median of daily trading price volume per year. Panel B reports the characteristics of the value-, volume- and equally-weighted market returns, bitcoin returns, ethereum returns and ripple returns. For returns, the mean, median and standard deviation are reported in percentage. Values are for the last day of each year with the exception of the year 2021 for which we take the last day of the sample, that is 31 October 2021. Data are daily from Cryptocompare.

4 Portfolios

This section presents the properties of portfolios for alternative sorting procedures. We start with value-weighted portfolios sorted by crypto market betas, co-skewness and co-kurtosis. Because of data availability, we could compute time-series of the market capitalization for only a subset of the cross-section of coins ($\approx$ 700). Table 4 presents the mean portfolio returns and standard errors. We obtain a declining cross-section of value-weighted portfolio returns in the case of portfolios sorted by β^{Sk}, as for volume-weighted portfolios. In contrast, for value-weighted portfolios we do not obtain a cross-section of returns for portfolios sorted by β^{Mkt} or β^{Kur}. For portfolios sorted by β^{Kur}, we do get an increasing cross-section from portfolio 1 to 4.

Table 5 presents the mean returns and standard errors for portfolios sorted by slope coefficients in regressions of each coin return on the realized bitcoin skewness ($\beta^{RSk,i}$), kurtosis ($\beta^{RKur,i}$) and semi-variance ($\beta^{RSv,i}$) estimated using a rolling window of 100 days. For the definition of the realized skewness and kurtosis we follow Amaya et al. (2015) and we construct them starting from the same tick-by-tick bitcoin price data we used to construct the realized variance measure. The realized semi-variance is obtained as the difference between the realized variances of positive and negative bitcoin returns. We expect that coins with higher $\beta^{RSk,i}$ and $\beta^{RSv,i}$, and lower $\beta^{RKur,i}$, are riskier and they should offer higher average returns. The results in Table Table 5 confirm this intuition, although the average excess return for the long/short portfolio based on realized skewness and kurtosis are not statistically different from zero. Figure 1 plots the time-series of these realized measures and highlights their large variability that likely affect the precision of the estimates.

Table 3: The determinants of the non-crypto premium

	Risk premium ($\Delta m_{t+1\|t}$)											
	Mkt_{US}	Mkt_{DE}	Mkt_{UK}	Mkt_{EM}	$Govt_{US}$	$Govt_{EU}$	$Corp$	Vix	Oil	$Gold$	$Dollar$	PC_1
$const$	−0.000	−0.002	−0.001	0.001	−0.000	−0.003	0.000	−0.007	−0.004	−0.000	−0.002	0.033
	[0.001]	[0.001]	[0.001]	[0.002]	[0.004]	[0.003]	[0.000]	[0.021]	[0.003]	[0.000]	[0.004]	[0.039]
	crypto factors											
$Goog_{Btc}$	−0.000	−0.004	−0.002	0.001	0.000	0.007	0.001	0.007	−0.001	−0.000	−0.007	0.019
	[0.001]	[0.003]	[0.001]	[0.004]	[0.007]	[0.008]	[0.001]	[0.035]	[0.003]	[0.000]	[0.009]	[0.088]
$Reddit_{Btc}$	−0.019	0.170^c	0.056	0.012	0.096	−0.167	−0.022	0.466	0.052	0.000	0.079	−1.016
	[0.045]	[0.099]	[0.053]	[0.163]	[0.233]	[0.261]	[0.031]	[1.546]	[0.207]	[0.004]	[0.288]	[3.092]
$rAmihud_C$	0.005	−0.006	−0.003	−0.013	−0.019	0.008	−0.003	−0.124	0.036	0.001	0.061	0.082
	[0.005]	[0.017]	[0.006]	[0.016]	[0.029]	[0.033]	[0.004]	[0.209]	[0.024]	[0.001]	[0.039]	[0.359]
$rAmihud_J$	−0.007	0.033	0.015^c	0.005	0.022	0.037	0.002	0.116	−0.047	−0.001	−0.070	−0.462
	[0.009]	[0.021]	[0.009]	[0.027]	[0.058]	[0.056]	[0.006]	[0.404]	[0.040]	[0.001]	[0.081]	[0.591]
Mom_{BTC}	0.000	0.000	0.000	−0.000	−0.000	0.000	0.000	0.001	−0.000	0.000^c	0.000	−0.001
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.001]	[0.000]	[0.000]	[0.000]	[0.001]
	non-crypto factors											
Mkt	−0.017	−0.055	−0.042	0.035	0.325^b	0.133	−0.012	1.486^b	0.111	−0.001	−0.144	−1.476
	[0.041]	[0.069]	[0.031]	[0.060]	[0.154]	[0.149]	[0.019]	[0.668]	[0.083]	[0.002]	[0.177]	[1.366]
Smb	−0.000	0.004	−0.045	−0.107	0.104	0.099	0.003	0.260	0.082	−0.004	0.035	0.058
	[0.040]	[0.088]	[0.040]	[0.129]	[0.232]	[0.258]	[0.021]	[1.493]	[0.159]	[0.003]	[0.281]	[3.008]
Hml	0.055	−0.032	0.079^c	-0.148^c	−0.369	0.021	0.018	−0.948	0.406^b	−0.004	−0.134	1.385
	[0.050]	[0.074]	[0.042]	[0.090]	[0.234]	[0.260]	[0.020]	[1.073]	[0.204]	[0.003]	[0.265]	[2.605]
Mom	0.000	−0.000	0.000	−0.001	-0.003^c	−0.001	0.000	−0.003	0.001	−0.000	-0.005^b	0.023
	[0.000]	[0.001]	[0.000]	[0.001]	[0.002]	[0.002]	[0.000]	[0.008]	[0.001]	[0.000]	[0.002]	[0.019]
Vix	0.002	−0.008	−0.005	0.004	0.033	0.009	−0.001	0.191^b	0.000	0.000	−0.039	0.004
	[0.004]	[0.009]	[0.006]	[0.010]	[0.021]	[0.022]	[0.002]	[0.092]	[0.014]	[0.000]	[0.025]	[0.229]
$Gold$	−0.003	-0.101^c	−0.043	−0.139	0.117	0.078	−0.015	−0.783	0.012	−0.002	−0.000	1.381
	[0.033]	[0.058]	[0.036]	[0.088]	[0.182]	[0.183]	[0.015]	[0.949]	[0.100]	[0.002]	[0.217]	[1.978]
$Infl$	−0.008	−0.025	−0.006	−0.036	−0.078	−0.012	−0.005	-0.462^c	−0.035	−0.000	0.021	1.470^b
	[0.010]	[0.028]	[0.013]	[0.026]	[0.069]	[0.051]	[0.005]	[0.266]	[0.048]	[0.001]	[0.067]	[0.656]
R^2 (%)	-0.432	-0.273	0.634	-0.880	0.321	-0.078	-0.562	-0.839	4.083	0.578	0.878	-0.570

Notes: The table reports the results of time-series regressions of the first differences (Δ) in the risk premium ($\Delta m_{t+1|t}$) of the *Intercept* model on a set of crypto and non-crypto determinants observed at time t, i.e., at the time the conditional moments are formed. Standard errors in brackets are Newey and West (1987). The last row reports adjusted R-squares in percentages. The crypto determinants are: the daily volatility of the number of daily Reddit posts about bitcoin ($Reddit_{Btc}$); the daily change in the Google Trend Index for the query "Bitcoin" across all geographical areas ($Goog$); the daily ?'s illiquidity measure based on the realized standard deviation, respectively, associated to the continuous component of volatility ($rAmihud_C$) and to the jump term ($rAmihud_J$); the BTC momentum factor (Mom_{BTC}), defined as the cumulated BTC excess return between $t-10$ and $t-2$. The non-crypto determinants are the Fama-French three factors (Mkt, Smb, Hml); the returns on the VIX volatility index (Vix) and the gold price index ($Gold$); the change in the 5-year U.S. inflation breakeven rate ($Infl$). Data are daily for the period January 1 2017 to December 31 2020 from Kraken, Bloomberg, the Kenneth French's data library, Google, and Bitcoinity.

Table 4: Crypto portfolios (value-weighted)

	Portfolio returns					
	Low	2	3	4	*High*	**5 − 1**
Panel A:			Sorted by $\beta^{Mkt,i}$			
Mean	0.52	0.53	0.82	0.39	0.52	−0.00
SE	[0.11]	[0.15]	[0.14]	[0.14]	[0.14]	[0.14]
Panel B:			Sorted by $\beta^{Sk,i}$			
Mean	0.71	0.61	0.49	0.55	0.39	-0.32^b
SE	[0.16]	[0.14]	[0.13]	[0.13]	[0.12]	[0.16]
Panel C:			Sorted by $\beta^{Kur,i}$			
Mean	0.50	0.58	0.63	0.65	0.47	−0.03
SE	[0.13]	[0.13]	[0.14]	[0.14]	[0.14]	[0.15]

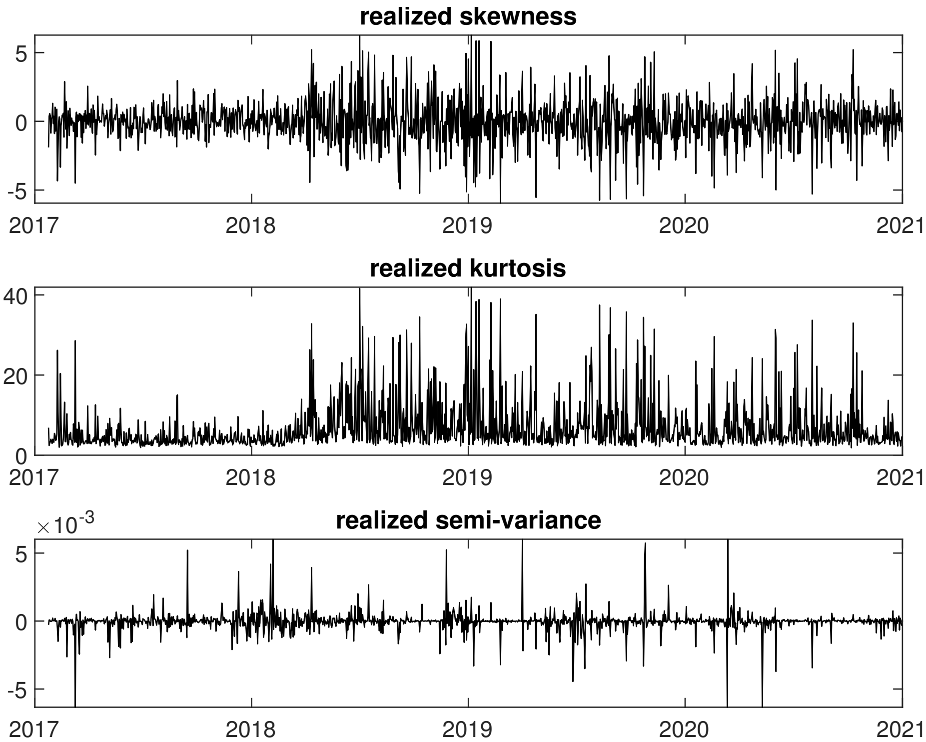
Notes: The table presents the mean daily returns, in percentage, of 5 portfolios sorted by crypto market betas (Panel A), co-skewness (Panel B) and co-kurtosis (Panel C). The table also reports, in brackets, standard errors by bootstrap. Portfolio 1 (*Low*) contains on average coins with the lower betas, while portfolio 5 (*High*) contains on average coins with higher betas. The last column reports the return of the zero-cost portfolio long in portfolio 5 and short in portfolio 1 (5 − 1). The coin-βs are estimated using a rolling window of 100 days. Portfolio returns are value-weighted (for details see Section 4.5). We denote with a, b, c superscripts significance at the 1%, 5% and 10% levels. Data are from Cryptocompare for the period January 1 2017 to December 31 2020.

Table 5: Crypto portfolios (sorted by realized higher-moments)

	Portfolio returns					
	Low	2	3	4	*High*	**5 − 1**
Panel A:			Sorted by $\beta^{RSk,i}$			
Mean	0.03	0.21	0.32	0.19	0.24	0.21
SE	[0.12]	[0.15]	[0.15]	[0.15]	[0.16]	[0.15]
Panel B:			Sorted by $\beta^{RKur,i}$			
Mean	0.18	0.27	0.22	0.25	0.10	−0.08
SE	[0.16]	[0.16]	[0.15]	[0.13]	[0.15]	[0.17]
Panel C:			Sorted by $\beta^{RSv,i}$			
Mean	0.01	0.23	0.24	0.21	0.28	0.26^c
SE	[0.13]	[0.14]	[0.13]	[0.15]	[0.17]	[0.16]

Notes: The table presents the mean daily returns, in percentage, of 5 portfolios sorted by slope coefficients in regressions of each coin return on the realized bitcoin skewness, kurtosis and semi-variance estimated using a rolling window of 100 days. The table also reports, in brackets, standard errors by bootstrap. Portfolio 1 (*Low*) contains on average coins with the lower betas, while portfolio 5 (*High*) contains on average coins with higher betas. The last column reports the return of the zero-cost portfolio long in portfolio 5 and short in portfolio 1 (5 − 1). The coin-βs are estimated using a rolling window of 100 days. Portfolio returns are value-weighted (for details see Section 4.5). We denote with a, b, c superscripts significance at the 1%, 5% and 10% levels. Data are from Cryptocompare for the period January 1 2017 to December 31 2020.

Figure 1: Realized measures



Notes: The figure plots the time-series for the realized skewness, kurtosis and semi-variance. Data are from Cryptocompare for the period January 1 2017 to December 31 2020.

5 Cross-sectional asset pricing

This section presents the procedures used to estimate the market prices of the crypto market, skewness and kurtosis risk.

5.1 Methodology

Linear factor models predict that average returns on a cross-section of assets can be attributed to risk premia associated with their exposure to a small number of risk factors. In the arbitrage pricing theory (APT) of Ross (1976), these factors capture common variation in individual asset returns. Table 6 illustrates the results of a principal component analysis of the panel of 15 portfolios, which reveals that four factors explain approximately 80% of the variation in portfolio returns. We note that for the portfolios sorted by β^{Mkt} and β^{Kur} the loadings corresponding to the first principal component increase from the first to last portfolio, and decrease for the second principal component for all portfolios.

Table 7 presents descriptive statistics of the three risk factors: Mkt_{Crypto}, Sk_{Crypto} and Kur_{Crypto}. The table shows that the crypto (value-weighted) market return is highly related to the bitcoin return (0.95) and the skewness associated with the crypto market return is negative (-0.32). We also note that the Kur_{Crypto} is positively correlated with the market return (0.38), while the Sk_{Crypto} has a correlation of zero with the market.

Table 8 reports the sample correlation coefficients of the three factors Mkt_{Crypto}, Sk_{Crypto} and Kur_{Crypto} with the first four principal components extracted by the panel of 15 portfolios. We note that the Mkt_{Crypto} is highly related to the first principal component (0.86). The Sk_{Crypto} is highly related to the fourth principal component (-0.84), while the Kur_{Crypto} is highly related to

Table 6: Portfolio principal components

Portfolio	1	2	3	4
	portfolios sorted by β^{Mkt}			
Low	0.12	0.63	-0.17	-0.23
2	0.25	0.09	0.39	0.07
3	0.27	-0.01	0.17	-0.04
4	0.27	-0.11	0.06	-0.09
High	0.31	-0.26	-0.34	0.00
	portfolios sorted by β^{Sk}			
Low	0.29	0.25	-0.25	0.84
2	0.30	0.03	0.27	-0.07
3	0.27	-0.07	0.17	-0.09
4	0.23	-0.10	0.05	-0.16
High	0.23	-0.10	-0.34	-0.31
	portfolios sorted by β^{Kur}			
Low	0.14	0.59	-0.18	-0.27
2	0.27	0.09	0.38	0.11
3	0.25	-0.06	0.11	-0.02
4	0.27	-0.10	0.04	-0.09
High	0.31	-0.25	-0.45	-0.03
% Var.	59.79	9.12	6.13	5.24

Notes: This table reports the principal component coefficients of the four components extracted from the panel of 15 portfolios sorted by crypto market (β^{Mkt}), skewness (β^{Sk}) and kurtosis (β^{Kur}) betas. The last row reports (in %) the share of the total variance explained by each of the first four principal components. Data are daily, from Cryptocompare. The sample period is January 1 2017 to December 31 2020.

Table 7: Descriptive Statistics Risk factors

Factors	Mkt_{Crytpo}	Sk_{Crypto}	Kur_{Crypto}
Mean (%)	0.44	-0.61	0.32
Std (%)	3.95	7.63	7.75
Skew	-0.32	-0.59	0.16
Kurt	8.40	9.08	10.52
$Corr(x, BTC)$	0.95	0.00	0.38

Notes: This table reports descriptive statistics of the three risk factors: Mkt_{Crypto}, Sk_{Crypto} and Kur_{Crypto}. Specifically, the table reports mean and standard deviation (in percentages), skewness and kurtosis. The last two reports the sample correlation coefficient with the bitcoin return. Data are daily, from Cryptocompare. The sample period is January 1 2017 to December 31 2020.

the second principal component (-0.79).

Table 8: Correlation matrix factors and principal comonents

$\rho(x,y)$	Mkt_{Crypto}	Sk_{Crypto}	Kur_{Crypto}
$PC1$	0.86	-0.17	0.39
$PC2$	-0.13	-0.34	-0.79
$PC3$	-0.04	-0.07	-0.21
$PC4$	-0.13	-0.84	0.17

Notes: This table reports the sample correlation coefficients of the three factors Mkt_{Crypto}, Sk_{Crypto} and Kur_{Crypto} with the first four principal components extracted by the panel of 15 portfolios. Data are daily, from Cryptocompare. The sample period is January 1 2017 to December 31 2020.

We use Rx^k_{t+1} to denote the average excess return in levels on portfolio k in period $t+1$. All asset pricing tests are run on excess returns in levels, not log excess returns, to avoid having to assume joint log-normality of returns and the pricing kernel. In the absence of arbitrage opportunities, this excess return has a zero price and satisfies the following Euler equation

$$E_t\left[M_{t+1}Rx^k_{t+1}\right] = 0.$$

We assume that the stochastic factor M is linear in the pricing factors Φ is such that

$$M_{t+1} = 1 - b\left(\Phi_{t+1} - \mu_\Phi\right),$$

where b is the vector of factor loadings, and μ_Φ denotes the factor means. This linear factor model implies a beta pricing model: the expected excess return is equal to the factor prices ι times the beta of each portfolio β^k, that is

$$E\left[Rx^j\right] = \iota'\beta^j,$$

where $\lambda = \Sigma_{\Phi\Phi}b$, and $\Sigma_{\Phi\Phi}$ is the variance-covariance matrix of the factors

$$\Sigma_{\Phi\Phi} = E\left(\Phi_t - \mu_\phi\right)\left(\Phi_t - \mu_\Phi\right)'.$$

The term β^k denotes the regression coefficients of the return Rx^k on the factors. To estimate the factor prices λ and the portfolio betas β, we use two different procedures: a Generalized Method of Moments estimation (GMM) applied to linear factor models, following Hansen (1982), and a two-state OLS estimation following Fama and MacBeth (1973), henceforth FMB. In the first step, we run a time series regression of returns on the factors. In the second step, we run a cross-sectional regression of average returns on the betas. We do not include a constant in the second step ($\lambda_0 = 0$) and therefore assume that assets with a beta equal to zero must offer zero excess returns.

5.2 Pricing alternative portfolios

In the paper, we build portfolios sorted by crypto market, co-skewness and co-kurtosis; then we extract a crypto skewness and kurtosis factor from these portfolios; and, finally, we show that a three-factor model with the crypto market, skewness and kurtosis factors can explain a large fraction of the cross-sectional variation in portfolio returns. A natural concern is related to the fact that we extract factors from test assets (although, this is fairly standard practice in the asset pricing literature). We address this concern by using the same three-factor model to price a different set of test assets.

We follow Liu et al. (2021), and build four additional sets of 5 portfolios starting from the same large panel of cryptocurrencies and the same time-frame used in our main analysis. Specifically, we build portfolios sorted by size, momentum, volume and volatility. For the size portfolios, we sort coins by their relative market size; for the momentum portfolios we sort coins by the past week return; for the volume portfolios we sort coins by the average daily trading volume in the previous week times price; for the volatility portfolios we sort coins by the average absolute daily return divided by price volume. All portfolios are rebalanced daily, using only information up to day t to sort coins, and then we compute returns from t to $t+1$.

Table 9 presents the asset pricing results for these 20 portfolios. The factors are the same we used in the main analysis; that is, the Mkt_{Crypto}, Sk_{Crypto} and Kur_{Crypto} factors. We note that, also in this case, the market prices of risk for the Mkt_{Crypto}, Sk_{Crypto} factors are significantly different from zero and very close to the respective sample means. In contrast, the market price of the Kur_{Crypto} is not statistically different from zero. The adjusted-R^2 are around 45%, and the pricing errors are around 40 basis points per day. These pricing errors are twice as large those of the main analysis.

Table 9: Asset pricing with alternative portfolios

	Risk Prices								
	$\iota_{Mkt_{Crypto}}$	$\iota_{Sk_{Crypto}}$	$\iota_{Kur_{Crypto}}$	$b_{Mkt_{Crypto}}$	$b_{Sk_{Crypto}}$	$b_{Kur_{Crypto}}$	R^2	$RMSE$	$\chi^2(\%)$
GMM_1	0.91	-0.62	0.11	4.67	-0.82	-0.71	46.63	0.39	
	[0.13]	[0.19]	[0.19]	[0.63]	[0.32]	[0.33]			0.00
GMM_2	1.00	-0.81	-0.10	5.31	-1.09	-1.17	43.02	0.41	
	[0.12]	[0.18]	[0.18]	[0.61]	[0.31]	[0.32]			0.00
FMB	0.91	-0.62	0.11	4.66	-0.82	-0.71	43.91	0.39	
	[0.11]	[0.18]	[0.18]	[0.55]	[0.31]	[0.33]			0.00
$Mean$	**0.88**	**-0.62**	**0.32**						
SE	[0.10]	[0.19]	[0.18]						

Notes: The table reports results from GMM and Fama and MacBeth (1973) asset pricing procedures on the 20 crypto portfolios sorted by size, momentum, volume and volatility. Market prices of risk ι, the adjusted R^2, the square-root of the mean-squared errors $RMSE$ and the p-values of χ^2 tests on pricing errors are reported in percentage points. b denotes the vector of factor loadings. All excess returns are daily and multiplied by 100. Shanken (1992)-corrected standard errors are reported in parentheses. We do not include a constant in the second step of the FMB procedure. We include the risk factors as additional test assets to impose the no-arbitrage conditions from the Euler equation. Data are daily, from Cryptocompare. The sample period is January 1 2017 to December 31 2020.

5.3 Tradable risk factors

We build the Sk_{Crypto} and Kur_{Crypto} factors as long/short strategies applied to portfolios sorted by co-skewness and co-kurtosis betas. In this section, we imagine actually implementing the strategies by presenting the asset pricing results in the case we replace these two factors with factor mimicking portfolios that are actually available to investors. The factor mimicking portfolios are constructed as a constrained linear projection of the long/short portfolios on a large set of portfolios: that is, the 15 portfolios which we use as test assets in the baseline specification along the alternative portfolios presented in Section 5.2 (that is, 4 sets of 5 portfolios sorted by size, momentum, volume and volatility). The factors mimicking portfolios, which we denote as $\widetilde{Sk}_{Crypto}$ and $\widetilde{Kur}_{Crypto}$, are zero-cost portfolios with portfolio weights $w_k \geq 0$, where $k = 1, \ldots, K$, on K portfolios; weight $w_{Btc} \leq 0$ on the bitcoin-to-dollar pair; and $\sum_k w + w_{Btc} = 0$. Therefore, the factor mimicking portfolios can only go long on any portfolio, and short only the bitcoin-to-dollar pair. We bound the long and short position to a leverage factor of 10 (results are not very sensitive to this parameter). The strategy of using a short position on the bitcoin pair follows the analysis in Borri and Shakhnov (2021) and is motivated by the fact that the short position on the bitcoin-to-dollar pair was always available to investors in our sample. Therefore, $\widetilde{Sk}_{Crypto}$ and $\widetilde{Kur}_{Crypto}$ can be considered tradable risk factors.

We first note that the factor-mimicking portfolios are not able to capture the same variability of the Sk_{Crypto} and Kur_{Crypto}. In fact, the R^2 we obtain by regressing the Sk_{Crypto} and Kur_{Crypto} on the two factor-mimicking portfolios are respectively 35% and 48%. The availability of options would most likely improve the fit of the factor-mimicking portfolios, but these are not available over the full sample. Despite the relatively low R^2, the three-factor model with the Mkt_{Crypto} factor and the two factor-mimicking portfolios performs reasonably well in pricing the cross-section of portfolio returns, where the test assets are the 15 crypto portfolios sorted by market, co-skewness and co-kurtosis betas. Table 10 presents our results. Only the market price of risk for the Mkt_{Crypto} is statistically significant; the market prices of the Sk_{Crypto} and Kur_{Crypto} are estimated less precisely but have the same sign, and are close in value, to the respective sample mean. The adjusted-R^2 is around 45% and the pricing errors around 15 basis points per day.

Table 10: Asset pricing with tradable risk factors

	Risk Prices								
	$\iota_{Mkt_{Crypto}}$	$\iota_{\tilde{Sk}_{Crypto}}$	$\iota_{\tilde{Kur}_{Crypto}}$	$b_{Mkt_{Crypto}}$	$b_{Sk_{Crypto}}$	$b_{Kur_{Crypto}}$	R^2	$RMSE$	$\chi^2(\%)$
GMM_1	0.97	-0.09	0.14	4.91	0.07	-1.35	54.90	0.14	
	[0.14]	[0.13]	[0.16]	[0.70]	[0.69]	[0.65]			0.08
GMM_2	1.03	0.00	0.22	5.12	0.52	-1.29	46.31	0.15	
	[0.14]	[0.12]	[0.15]	[0.66]	[0.61]	[0.57]			0.10
FMB	0.97	-0.09	0.14	4.91	0.07	-1.34	50.86	0.14	
	[0.11]	[0.12]	[0.15]	[0.57]	[0.68]	[0.65]			0.20
Mean	**0.97**	**-0.21**	**0.14**						
SE	[0.11]	[0.11]	[0.13]						

Notes: The table reports results from GMM and Fama and MacBeth (1973) asset pricing procedures on the 15 crypto portfolios sorted by market, co-skewness and co-kurtosis betas. Market prices of risk ι, the adjusted R^2, the square-root of the mean-squared errors $RMSE$ and the p-values of χ^2 tests on pricing errors are reported in percentage points. b denotes the vector of factor loadings. All excess returns are daily and multiplied by 100. Shanken (1992)-corrected standard errors are reported in parentheses. We do not include a constant in the second step of the FMB procedure. Data are daily, from Cryptocompare. The sample period is January 1 2017 to December 31 2020.

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